ABSTRACTS OF ARTICLES DEPOSITED AT VINITI*

MOTION OF AEROSOL PARTICLES HEATED BY INTERNAL SOURCES IN EXTERNAL FIELDS OF TEMPERATURE AND CONCENTRATION GRADIENTS

Yu. I. Yalamov, M. F. Barinova,
A. V. Myagkov, and E. R. Shchukin

UDC 533.72

A theory is constructed for the motion of spherical particles in gaseous media under the action of forces resulting from a nonuniform distribution of temperature and concentration, such a nonuniformity being caused by arbitrarily oriented external temperature and concentration gradients as well as by internal heat sources. Expressions are derived for the total force acting from the gas on an aerosol particle and for the rate at which the motion of such a particle stabilizes. Methods are shown by which the velocity of internally heated particles and the forces acting on them, in any external field of temperature and concentration gradients, can be calculated. In the case of a spherical particle moving due to internal heating only, an analysis of the process includes also the causes of photophoresis.

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EXPERIMENTAL STUDY CONCERNING THE EFFECT OF COMPLIANT SURFACES ON THE INTEGRAL CHARACTERISTICS OF A BOUNDARY LAYER

V. I. Korobov

UDC 532.526-597.31

Studies have already been made concerning the interaction of compliant boundaries with an oncoming stream, these boundaries being in most cases between air and membrane surfaces. The view prevails, based on hydrobionic studies, that elastic surfaces such as integument play an active functional role in the hydrodynamics of aquatic animals.

Here results are presented of an experimental study concerning the hydrodynamic friction at compliant monolithic surfaces, taking into account the morphofunctional structure of external integuments in aquatic animals. These experiments were performed in a water flume with an insert on a tensometric mounting in the lower wall of the test segment.

The main results of this study can be formalized as follows.

Compliant surfaces can have a significant effect on the integral characteristics of a boundary layer and a compliant surface with constant mechanical characteristics has, moreover, an effect on a boundary layer within a specific relatively narrow range of the Reynolds number with the situation and the width of the peak within that range depending on several parameters of the surface.

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*All-Union Institute of Scientific–Technical Information.

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EFFECT OF "WHITE NOISE" ON THE PROCESS OF INERTIAL SEPARATION

E. A. Eseev

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The stochastic differential equation

$$\frac{dr}{dt}=\tau\left(1-\frac{\rho}{\rho_1}\right)\frac{v_\varphi^2}{r}+g\xi(t) \tag{1}$$

is applicable [1] to transient turbulent motion of a whirled disperse system where trajectories of all particles of the carrier medium have been assumed to be arcs of concentric circles and the movement of the suspensions (Stokes fractions) relative to the dispersing medium has been assumed to be quasisteady. As is well known [2], to Eq. (1) corresponds the equation

$$\frac{\varepsilon g^2}{2}\frac{d^2f(r_0)}{dr_0}+\tau\left(1-\frac{\rho}{\rho_1}\right)\frac{v_\varphi^2}{r_0}\frac{df(r_0)}{dr_0}=-1. \tag{2}$$

Letting v_φ = const r_0^γ and approximating some profile of tangential velocities of the carrier medium with $\gamma = 0.5$, we can rewrite Eq. (1) in dimensionless form as

$$\alpha\frac{d^2\bar{f}(\bar{r}_0)}{d\bar{r}_0^2}+\beta\frac{d\bar{f}(\bar{r}_0)}{d\bar{r}_0}=-1, \tag{3}$$

where α and β are the dimensionless coefficients of diffusion and drift, respectively.

The solution to Eq. (2) for the boundary conditions

$$\frac{d\bar{f}(\bar{R}_1)}{d\bar{r}_0}=0,\quad \bar{f}(\bar{R}_2)+\lambda\frac{d\bar{f}(\bar{R}_2)}{d\bar{r}_0}=0, \tag{4}$$

is

$$\bar{f}(\bar{r}_0)=\frac{\bar{R}_2-\bar{r}_0}{\beta}-\frac{\alpha}{\beta^2}\left\{\exp\left[-\frac{\beta}{\alpha}(\bar{r}_0-\bar{R}_1)\right]-\exp\left[-\frac{\beta}{\alpha}(\bar{R}_2-\bar{R}_1)\right]\right\}+\frac{\lambda}{\beta}\left\{1-\exp\left[\frac{\beta}{\alpha}(\bar{R}_2-\bar{R}_1)\right]\right\}. \tag{5}$$

Here $\bar{R}_1$ and $\bar{R}_2$ denote, respectively, the inside boundary and the outside boundary of the separation zone, and parameter λ characterizes the intensity of breakaway of particles of some fraction from boundary $\bar{R}_2$. In expression (5) the first term is always positive, the second term is always negative, and the third term is determined by parameter λ.

An inspection as to whether the same solution (5) holds true also for some other empirical v_φ profiles, specifically for $\gamma = 1$ and $\gamma = -1$, made by numerical integration of the forward Fokker–Planck–Kolmogorov equation for process $\bar{r}(\bar{t})$ with a probability density $w(\bar{r}, \bar{t})$ and conditions at $\bar{R}_1$, $\bar{R}_2$ analogous to boundary conditions (4), has revealed a complete agreement of the results with solution (5). As the value of coefficient α increases, the calculated fraction efficiencies at corresponding instants of time $\bar{t}$ within the process period increase and the shape of the v_φ profile does not influence this trend in any way.

NOTATION

r, φ	are the polar coordinates;
t	is the time;
v_φ	is the tangential component of the velocity field in the dispersing medium;
τ	is the relaxation time of a separating particle;
ρ, ρ_1	are the physical densities of the medium and of a particle, respectively;
g	is the parameter characterizing a specific dispersion system;
ξ, ε	are the "white noise" and its intensity;
f	is the average time for a particle to reach the boundary;

r_0 is the initial distribution of the polar radius;

$\bar{r}_0 = 2r_0/(R_1 + R_2)$ is the $\bar{f}$ etc., dimensionless characteristics of corresponding quantities.

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STUDY OF THE P−ρ−T RELATION FOR 2,3-BUTYLENE GLYCOL OVER WIDE RANGES OF TEMPERATURE AND PRESSURE

T. A. Apaev, I. G. Imanova, and A. M. Kerimov

UDC 536.2

The density of liquid 2,3-butylene glycol was measured for the first time over the temperature range from 290 to 620°K and the pressure range from 1 to 800 bar by the method of hydrostatic weighing. Some of the experimental P−ρ−T data are tabulated here.

The maximum relative error of density determination, including possible measurement and conversion errors, was ±0.1%.

Processing and evaluation of the P−ρ−T data for 2,3-butylene glycol were done according to the A. M. Kerimov−T. A. Apaev method, which these authors had proposed for liquid hydrocarbons. The sections across isotherms at constant-density coordinates were found to be straight lines, for 2,3-butylene glycol too, accurately within ±0.12% of density values. For describing the P−T relation at ρ = const, accordingly, we used the equation

$$p = A(\rho) + B(\rho)T, \qquad (1)$$

with A and B both fractions of ρ, P denoting the pressure on the liquid (bar), and T denoting the temperature (°K).

On the basis of rigorous thermodynamic analysis, the coefficients in Eq. (1) can be shown to have the physical significance of

$$A = -\left(\frac{\partial \varphi}{\partial V}\right)_T, \quad B = \left(\frac{\partial P}{\partial T}\right)_v. \qquad (2)$$

TABLE 1. Density of 2,3-Butylene Glycol, $\rho \cdot 10^{-3}$ kg/m^3

P, bar	T, °K						
	291,37	350,27	408,15	469,92	513,99	568,15	623,15
1,0123	0,9962	0,9484	0,8985	—	—	—	—
99,012	1,0014	0,9547	0,9059	0,8458	0,7944	0,7212	0,6070
197,01	1,0062	0,9606	0,9132	0,8570	0,8100	0,7451	0,6618
295,01	1,0110	0,9667	0,9205	0,8674	0,8239	0,7648	0,6946
393,01	1,0155	0,9724	0,9277	0,8770	0,8364	0,7808	0,7191
491,01	1,0194	0,9780	0,9345	0,8859	0,8474	0,7946	0,7391
589,01	1,0232	0,9834	0,9412	0,8948	0,8580	0,8069	0,7557
687,01	1,0268	0,9881	0,9479	0,9037	0,8677	0,8183	0,7696
785,01	1,0302	0,9924	0,9546	0,9126	0,8772	0,8294	0,7824

Here φ is the energy of intermolecular interaction and V is the volume. Expression (2) can easily be reduced to the form $A = -d\varphi/dV$ and, therefore, $d\varphi = -AdV$, since $V = \mu v$, with μ denoting the molecular mass and $v = 1/\rho$ denoting the specific volume of the liquid at given values of the state parameters, so that

$$d\varphi = -\mu A(v)\,dv. \tag{3}$$

Inasmuch as the curve of $A = f(v)$ asymptotically approaches the v axis, integrating the expression (3) from a fixed specific volume v_0 to ∞ will yield the energy of intermolecular interaction

$$\varphi = -\mu \int_{v_0}^{\infty} A(v)\,dv.$$

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DIELECTROMETRIC INSPECTION AND CONTROL OF HIGH-FREQUENCY DRYING OF HETEROGENEOUS MATERIALS

N. V. Sedykh and L. G. Sedykh

UDC 66.047.354

With a view toward optimization of the drying process in microwave fields, an extremal problem mathematically formalizing this process is considered here.

In order to solve this problem in linear programming, it is necessary to measure the dielectric parameters of the object being dried: its dielectric constant ε, loss tangent $\tan\delta$, and relaxation time τ.

An experimental determination of ε, $\tan\delta$, and τ during the drying process is not possible by classical methods because of the lengthiness of this process. Therefore, plotting the dielectric spectra [1] by the pulse method is proposed here for this purpose.

Into the system under consideration is transmitted a rectangular pulse signal whose Fourier spectrum extends over a very wide frequency range.

A Fourier analysis of the incident pulse as well as of the pulses, respectively, reflected by and transmitted through the object yields quantitative data about the frequency dependence of the dielectric properties of the material.

Using the dielectric parameters and approximately regarding the object being dried as a two-component mixture, one can calculate its moisture content according to the relations for dielectric mixtures [2]. As the selection criterion we use the relaxation time τ and the interval of time distribution.

Dielectrometric inspection makes it possible to regulate the drying process by varying either the power output or the frequency of the microwave oscillator.

The proper mode of regulation can be selected by a computer, upon checking the actual process conditions against the prescribed ones, on the basis of a simulation of the process in the form of the solution to the system of equations describing heat and mass transfer during microwave heating.

It is proposed that Pontryagin's principle of the maximum [3] be applied to the forming of control signals which will optimally fast veer the system (object being dried) to the prescribed condition.

An analysis of the proposed algorithms is made here for the case of drying a yeast suspension in microwave fields at optimum frequencies, with current correction of the process to optimality. Application of this method has made it possible to accelerate the drying process and render it more economical.

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RADIATIVE HEAT TRANSFER IN A COAXIAL SYSTEM OF TWO CONTINUOUS CYLINDERS WITH A PERFORATED ONE BETWEEN THEM

A. V. Rumyantsev, O. N. Bryukhanov, and V. R. Bazilevich

UDC 536.24

The net fluxes of radiation energy flowing between the elements of a system which consists of three infinitely long coaxial cylinders, the middle cylinder with holes in the surface, are determined in the diffusion approximation from given temperatures and opticogeometric characteristics. The problem is solved by the general zonal method, for which expressions are derived describing the mean angular coefficients of radiation:

$$\varphi_{14}=\beta,\quad \varphi_{12}=1-\beta,\quad \varphi_{22}=(1-\zeta_{12})(1-\beta),$$
$$\varphi_{21}=\zeta_{12},\quad \varphi_{24}=\beta(1-\zeta_{12}),\quad \varphi_{42}=\zeta_{24}(1-\beta)\varphi_{24},$$
$$\varphi_{41}=\beta\zeta_{14},\quad \varphi_{43}=(1-\beta)\zeta_{24},\quad \varphi_{44}=1-\zeta_{24}[1-\beta^2(1-\zeta_{12})].$$

Here $\zeta_{ik} = D_i/D_k$ are the ratios of diameters of the cylindrical surfaces and β is the perforation index of surface F_2. Subscripts 1 and 3 refer to convex surfaces, subscripts 2 and 4 refer to concave surfaces.

A numerical evaluation of the calculated results reveals a nonlinear dependence of the net energy fluxes radiated by surfaces F_2 and F_4 on the parameter β. The energy flux from surface F_1 increases monotonically with increasing β, which can be explained by a decrease in the shielding effect of the perforated cylinder. The energy flux radiated by surface F_3 decreases linearly with increasing β, owing to a decrease in the surface area. The energy fluxes radiated by surfaces F_2 and F_4 depend nonlinearly on the variable β.

The function describing the net flux from surface F_2 to surface F_4 has a maximum which shifts toward lower values of β, as the emissivity ε decreases. The anomalous radiation characteristic of this surface is due to three differently influencing causes: 1) perforation (cavity) effect; 2) change in the magnitude of the radiation flux impinging on the receiver surface as the perforation area changes; and 3) change in the area of the emitter surface as β changes. These factors make this function nonlinear with a maximum.

The perforated middle cylinder can be regarded as shield with holes between the two continuous cylinders. A comparison of energy fluxes impinging on surface F_4 with the presence of respectively a perforated or continuous cylinder in the system at the same temperatures in each case, indicates their ratio is larger than unity for any values of ε and β. The difference between the magnitudes of energy fluxes increases with lower ε values and this trend is attributable to the perforation effect, which significantly influences the pattern of radiation from surface F_2.

These theoretical data also yield the energy radiated in a system of coaxial cylinders with a continuous one inside and a perforated one outside.

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ANGULAR COEFFICIENTS FOR A PARTIALLY SHIELDED CYLINDRICAL SURFACE

V. A. Arkhipov

UDC 536.3

The angular radiation coefficients for the surfaces of a disk and of an infinitely long cylinder, separated by a shield parallel to the disk and having a diaphragm coaxial with it, are calculated by the method of numerical integration, assuming that the normal to the disk at its center intersects the cylinder axis at right angles. Such a configuration is of interest, e.g., in the design of optoelectronic equipment for diagnosis of plasma jets with a laser. The mean angular coefficient is calculated according to the general relation for $\varphi_{1,2}$ written in a form applicable to this particular geometry:

$$\varphi_{1,2}=\frac{4R}{(\pi r_1)^2}\int\limits_0^{z_k} dz\int\limits_0^{\xi_k}(L-R\cos\xi)\,d\xi\int\limits_0^{r_1} r\,dr\int\limits_0^{2\pi}\frac{(r\sin\xi\sin\psi+L\cos\xi-R)\,d\psi}{(R^2+L^2+z^2+r^2-2RL\cos\xi-2zr\cos\psi-2rR\sin\xi\sin\psi)^2}.$$

For the variable integration limits $z_k(r, \psi, \xi)$, $\xi_k(r, \psi)$, which define the field of vision, the analytical expressions

$$z_k=r\cos\psi+\frac{L-R\cos\xi}{l}\left(\sqrt{r_2^2-\left(r\sin\psi+\frac{l\,(R\sin\xi-r\sin\psi)}{L-R\cos\xi}\right)^2}-r\cos\psi\right),$$

$$\xi_k=\arcsin\frac{Lr_2-(L-l)\,r\sin\psi}{R\sqrt{l^2+(r_2-r\sin\psi)^2}}-\operatorname{arctg}\frac{r_2-r\sin\psi}{l}$$

are obtained.

On the basis of the algorithm set up here, calculations are made and graphs are plotted for mean and local (with the disk elongated into an elementary area) angular coefficients corresponding to various values of the geometric parameters of the given radiation system.

NOTATION

l is the distance between disk and shield;
L is the distance from the disk to the cylinder axis;
r, ψ are the cylindrical coordinates of a point on the disk surface;
r_1 is the disk radius;
r_2 is the diaphragm radius;
R is the cylinder radius;
z, ξ are the cylindrical coordinates of a point on the cylinder surface.

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HEAT TRANSFER BETWEEN A DOUBLE-LAYER PLATE AND AN EMITTING AND SCATTERING MEDIUM IN MOTION

F. N. Lisin and I. F. Guletskaya UDC 536.3

A gray emitting, absorbing, and scattering medium moves through a slot channel with gray walls. The bottom wall is a double-layer plate of a given thickness and with known thermal conductivities. The velocity profile is parabolic. The thermal flux at the lower surface of the double-layer plate is given as a function of the x coordinate, and the temperature of the top wall is given as constant T_W. In dimensionless form, the problem is written in the form of equation

$$N_{Pe}\frac{u(\eta)}{\bar{u}}\frac{\partial\theta}{\partial\bar{x}}=\frac{\partial^2\theta}{\partial\eta^2}-\frac{N_{Pe}}{N_{Bo}}\operatorname{div}\bar{q} \tag{1}$$

for the energy of the moving medium, and equation

$$\frac{\partial^2\theta_i}{\partial\bar{x}^2}+\frac{\partial^2\theta_i}{\partial\eta^2}=0,\ i=1,\ 2 \text{ denoting the respective layers,} \tag{2}$$

for the heat transfer within the layers of the plate, with the boundary conditions

$$\theta=1 \ \text{at}\ \bar{x}=0;\quad \theta=\theta_{CT}\ \text{at}\ \bar{x}>0;\quad \eta=1;$$

$$\frac{\partial\theta}{\partial\eta}-\frac{Pe}{Bo}\bar{q}^r_{CT}=k_1\frac{\partial\theta_1}{\partial\eta}\ \text{at}\ \eta=0;$$

$$\theta(\bar{x},0)=\theta_1(\bar{x},0);\quad \frac{\partial\theta_1}{\partial\bar{x}}=\frac{\partial\theta_2}{\partial\bar{x}}=0\ \text{at}\ \bar{x}=0,\ \bar{x}=L,$$

$$k_2\frac{\partial\theta_1}{\partial\eta}=\frac{\partial\theta_2}{\partial\eta},\quad \theta_1(\bar{x},\eta)=\theta_2(\bar{x},\eta)\ \text{at}\ \eta=-\bar{S}_1,$$

$$\frac{\partial\theta_2}{\partial\eta}=\bar{q}_2(\bar{x})\ \text{at}\ \eta=-\bar{S}_2,$$

where $\eta=y/b$; $\bar{x}=x/b$; $\bar{\theta}=T/T_0$; $k_1=\lambda_1/\lambda$; $k_2=\lambda_1/\lambda_2$; $\bar{S}_1=S_1/b$; $\bar{S}_2=S_2/b$; $\bar{q}^r=q^r(\bar{x})/\sigma_0T_0^4$.

The divergence of radiation flux is found from the solution to the transfer equation, in the P_1 approximation of the method of spherical harmonics, and it includes the mean scattering cosine. On the basis of a numerical solution, the dependence of the Nusselt number on the optical thickness is analyzed for radiative flux and convective flux to the plate and to the top wall. The quantity of heat transferred by radiation to the walls, as a function of $\tau_0(1-\gamma)$, passes through a maximum within the 1.1-1.2 range (τ_0 denoting the optical thickness and γ denoting the ratio of scattering coefficient to attenuation coefficient). On the basis of calculations is also analyzed the dependence of the heat-transfer rate on the ratio of thermal conductivities $k_2=\lambda_1/\lambda_2$. As k_2 increases, the temperature at the $\eta=0$ surface rises and this affects the cooling of the medium in the channel. The higher the value of k_2 is, the higher lie the curves of the radiation Nusselt number as a function of the channel length.

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EXAMINATION AND CALCULATION OF THE ENERGY CHARACTERISTICS OF RADIATIVE HEAT TRANSFER IN A RADIATION SYSTEM CONSISTING OF TWO COAXIAL CYLINDERS OF DIFFERENT LENGTHS AND SEPARATED BY AN ATTENUATING MEDIUM

Yu. A. Surinov and V. V. Rubtsov

UDC 536.3

The second variant and the third variant of the generalized zonal method according to Surinov [1-3] are applied to a numerical analysis and solution of the mixed problem of radiative heat transfer in a radiation system, in the case where the latter is bounded by two coaxial cylinders of different finite lengths and filled with an absorbing as well as isotropically scattering medium so that it can be regarded as a single isothermal volumetric zone at a given temperature. The boundary surface F of this system is subdivided into six zones (two lateral surfaces of the inner cylinder and the outer cylinder, respectively, also the two base surfaces of each). Given are the temperatures of the inner cylinder at its lateral and both base surfaces, also at one of the base surfaces of the outer cylinder, and the net radiative flux at the other base surface and the lateral surface of the outer cylinder. Determined are the surface densities of the net radiation flux at zones of given temperatures and the temperature fields of zones with given radiation fluxes, also the volume density of the net radiation flux and the spherical radiation vector at internal points of the system.

For the purpose of determining these energy characteristics of radiation, a preliminary numerical evaluation was made of the various opticogeometric resolving functions and, particularly, the generalized resolvent angular radiation coefficients $\Pi(M_i, F_k)$ as well as the generalized resolvent solid angles $\Pi^{(1)}(M, F_k)$ according to the expressions

$$\Pi(M_i, F_k) = \psi(M_i, F_k) + \sum_{j=1}^{6} \tilde{R}_j \Pi_{jk} \psi(M_i, F_j) + \frac{p}{4} \Pi^{(1)}(V, F_k) A(M_i, V);$$

$$(M_i \in F_i; \quad i, k = 1, 2, \ldots, 6);$$

$$\Pi^{(1)}(M, F_k) = \psi^{(1)}(M, F_k) + \sum_{j=1}^{6} \tilde{R}_j \Pi_{jk} \psi^{(1)}(M, F_j) + \frac{p}{4} \Pi^{(1)}(V, F_k) A^{(1)}(M, V); \ (M \in V).$$

The results of this numerical evaluation are presented in graphical form depicting the respective relations for the energy characteristics of radiation. Noteworthy are the dimensionless boundary characteristics of radiation as functions of the coordinates, obtained for various values of the Bouguer number. The effect of scattering by the medium on these radiation characteristics is also examined.

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APPROXIMATE ANALYSIS OF TEMPERATURE FIELDS IN COATINGS DURING ENAMELING OF CONDUCTORS

N. A. Tsvetkov and A. S. Lyalikov

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The problem of radiative – convective heat transfer involving conductors is formulated so as to apply to the technology of enameling from a melt in two extreme cases (an insulation coating respectively opaque and transparent to thermal radiation).

A wire and an enamel coating are represented as two cylindrical bodies, whereupon the relatively thin coating is assumed to be flat. Owing to low values of the Biot number for wire (copper, aluminum, constantan, manganin, and other metals), the latter could be regarded as a thermally thin body.

A numerical analysis of the resulting system of differential equations is limited in scope, because of the length of machine time, and therefore an approximate analytical solution is additionally obtained with the aid of a Laplace integral transformation. The sought fourth-degree temperature functions are, moreover, approximated by piecewise-linear functions with 20°C linear segments. The coefficients of this approximation, which have been calculated beforehand, are given in a table.

With the aid of these solutions, the influence is analyzed which the process conditions in the active chamber of a horizontal enameling oven have on establishment of the temperature level and on the magnitude as well as the direction of the temperature gradients across the coating thickness in the said two extreme cases.

It is demonstrated that in the case of a coating transparent to thermal radiation one can estimate the maximum rates of heat treatment with radiative – convective heat application to the wire – coating system.

The results of this study suggest that in enameling ovens for conductors treated in a melt it is expedient to establish two zones with separate regulation of the basic process parameters (temperature and velocity of the heat carrier, temperature at the wall surfaces of the active chamber as, e. g., in VRE-144 ovens of the Italian Sicme Co. for enameling from a solution).

From economic considerations and limitations on the capabilities of materials, one can determine the length of the first zone and prescribe its process parameters so that the temperature level in the conductor will reach 80-90% of the optimum temperature for insulation curing.

In the second zone must be established the optimum magnitude and the necessary direction of the temperature gradient across the coating thickness so that, while the optimum temperature level is maintained in the conductor, the insulation layer will cure properly.

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ASYMPTOTIC SOLUTION TO THE PLANE STEADY-STATE PROBLEM IN THE THEORY OF HEAT CONDUCTION WITH A BOUNDARY CONDITION OF THE THIRD KIND APPLIED TO A CYLINDRICAL PIPE BURIED IN SOIL

B. A. Vasil'ev

UDC 517.946

Let it be required to determine the steady-state temperature distribution between an infinitely long circular cylinder and a plane target to it, when the surface of the cylinder emits a uniformly distributed thermal flux and the plane surface is cooled according to Newton's law by a medium at zero temperature [1]. With the aid of a Fourier integral transform in a system of degenerate bipolar coordinates, this problem can be reduced to the equation

$$M''(\nu) - ha\,\frac{\operatorname{cth}\nu}{\nu}\,M(\nu) = -\frac{2\operatorname{sh}\nu}{\operatorname{ch}^3\nu}, \quad 0 < \nu < \infty, \tag{1}$$

to be solved for the boundary conditions

$$M(\nu) = O(\nu^{1+\varepsilon}), \ \nu \to +0, \ \varepsilon > 0, \ \lim_{\nu \to +\infty} M(\nu) = 0,$$

where $M(\nu)$ is an auxiliary function, unknown, and h is a positive constant; a is the cylinder diameter.

For low values of the parameter ha Eq. (1) reduces to a Fredholm integral equation of the second kind, which can be solved by the method of successive approximations. The kernel of the solution appears in terms of modified Bessel functions [2]. For high values of the parameter ha, the solution to Eq. (1) is sought in the form of an asymptotic series

$$M(\nu) = \sum_{k=1}^{\infty} M_k(\nu)\,(ha)^{-k}. \tag{2}$$

The terms in series (2) are successively determined from the recurrence relations

$$M_1(\nu) = \frac{2\nu\operatorname{sh}^2\nu}{\operatorname{ch}^4\nu}, \quad M_{k+1}(\nu) = \nu \operatorname{th}\nu M_k''(\nu).$$

After the solution to Eq. (1) has been found, the temperature distribution over the plane can be written as

$$T(\beta) = \frac{Q}{\pi K}\left\{\frac{1}{ha} - \int_0^\infty M(\nu)\,\frac{\operatorname{cth}\nu}{\nu}\,\cos\nu\beta\,d\nu\right\},$$

where K is the thermal conductivity; Q, thermal flux per unit time per unit length; and β, curvilinear coordinate of points on the tangent plane.

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CALCULATION OF TRANSIENT TEMPERATURES IN SYSTEMS OF PLATES AND BEAMS

V. F. Kravchenko and V. M. Yudin

UDC 536.24.02

Under consideration is the problem of heating a plane system of N beams and M rectangular plates joined at R nodes and along K lines, on the assumption that the system contains U closed cavities of arbitrary shape.

The thermophysical properties of the materials depend on the temperature, and the thermal conductivity of the materials of plates, moreover, may be anisotropic.

At the lateral surfaces of the beam as well as at the boundaries of the plates there can occur various thermal processes: aerodynamic action, convection, natural convection, radiative heating, heating by a given thermal flux, intrinsic radiation, radiative heat transfer through the internal cavities, also any physically possible combination of these processes.

The system of equations describing the propagation of heat through such a structure consists of N equations of heat conduction for the beams, M equations of heat conduction for the plates, and U systems of integral equations of radiative heat transfer through the internal generally nonconvex cavities.

The boundary conditions at the joining nodes and lines are stipulated in terms of equal temperatures of contiguous elements and zero sum of thermal fluxes at each joining node and line.

The problem is solved by the implicit scheme in the method of elementary heat balances. Inasmuch as the temperature dependence of the thermophysical properties is taken into account as well as intrinsic radiation and radiative heat transfer through the cavities, the system of difference equations is written for being solved by the iteration method. The accuracy of the solution is improved by approximating the boundary conditions at the ends of the beams in the second order [1]. The locally uniform scheme is used for difference approximation of the equations for the plates.

The system of integral equations of radiative heat transfer is algebraized with the aid of Markov quadratures, according to the method shown in [2].

As a result, on each time interval there appears a system of algebraic equations constituting a set of N tridiagonal subsystems for the temperatures of beam elements and 2M tridiagonal subsystems for the temperatures of plate elements, interconnected by equations for the temperatures of the joining nodes, also not connected with them and with one another U subsystems of equations for the density of incident thermal flux.

For the solution of this system there has been constructed an iteration process with respect to the temperatures of the joining nodes which also takes care of nonlinearities. The subsystems of equations of radiative heat transfer are solved by the method in [2] and the tridiagonal subsystems are solved by the elimination method.

A program in the FORTRAN language has been written on the basis of this algorithm and heating of a rectangular caisson is calculated consisting of a three-stringer upper panel and a smooth lower panel.

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DYNAMIC TEMPERATURE FIELDS IN HOMOGENEOUS HOLLOW SPHERICAL BODIES

V. V. Semenyuk and M. P. Lenyuk

UDC 536.21

The solution of classical problems in the theory of linear thermoelasticity requires that the structure of the true temperature field during highly nonsteady processes be known. The temperature field in an isotropic homogeneous hollow spherical body $D = \{(r, \theta, \varphi), R_1 \le r \le R_2, 0 \le \theta \le \pi, 0 \le \varphi \le 2\pi\}$ during such processes is described by the scalar quantity T which happens to be the solution to the generalized (hyperbolic) heat-conduction equation [1]

$$L[T] \equiv b_0^2 \frac{\partial^2 T}{\partial t^2} + b_1^2 \frac{\partial T}{\partial t} - a^2 \left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \left\{ \frac{\partial}{\partial \mu} \left[(1-\mu^2) \frac{\partial T}{\partial \mu} \right] + \frac{1}{1-\mu^2} \frac{\partial^2 T}{\partial \varphi^2} \right\} \right) = f_1(t, r, \mu, \varphi), \ \mu = \cos\theta \tag{1}$$

for the initial-boundary conditions

$$T|_{t=0} = f_2(r, \mu, \varphi), \quad \left.\frac{\partial T}{\partial t}\right|_{t=0} = f_3(r, \mu, \varphi), \tag{2}$$

$$\left(h_{j1} \frac{\partial}{\partial r} + h_{j2} \frac{\partial}{\partial t} + h_{j3} \right) T|_{r=R_j} = (-1)^{j+1} \psi_j(t, \mu, \varphi), \ j = 1, 2 \tag{3}$$

and the uniqueness conditions with respect to (φ, θ).

The solution to problem (1)-(3) is constructed with the aid of principal solutions (fundamental functions) of the problem in the form

$$\begin{aligned} T = & \int_0^t d\tau \int_0^{2\pi} d\alpha \int_{-1}^{+1} d\eta \int_{R_1}^{R_2} \mathrm{E}(t-\tau; r, \rho; \mu, \eta; \varphi, \alpha) \rho^2 f_1(\tau, \rho, \eta, \alpha)\, d\rho \\ & + \int_0^t d\tau \int_0^{2\pi} d\alpha \int_{-1}^{+1} [W_-(t-\tau; r, \mu, \eta; \varphi, \alpha) \psi_1(\tau, \eta, \alpha) + W_+(t-\tau; r, \mu, \eta; \varphi, \alpha) \\ & \times \psi_2(\tau, \eta, \alpha)]\, d\eta + \int_0^{2\pi} d\alpha \int_{-1}^{+1} d\eta \int_{R_1}^{R_2} K(t, r, \rho; \mu, \eta; \varphi, \alpha) \Big[f_3(\rho, \eta, \alpha) \\ & + \frac{b_1^2}{b_0^2} f_2(\rho, \eta, \alpha) \Big] \rho^2 d\rho + \frac{\partial}{\partial t} \int_0^{2\pi} d\alpha \int_{-1}^{+1} d\eta \int_{R_1}^{R_2} K(t, r, \rho; \mu, \eta; \varphi, \alpha) f_2(\rho, \eta, \alpha) \rho^2 d\rho. \end{aligned} \tag{4}$$

A significant role in the construction of fundamental functions E, K, and $W_\pm$ of problem (1)-(3) is played by the finite Legendre–Fourier integral transformations, forward Λ_{mn} and inverse Λ_{mn}^{-1},

$$\Lambda_{nm}[f(\varphi, \mu)] = \int_{-1}^{+1} \int_0^{2\pi} f(\varphi, \mu) e^{im\varphi} P_n^m(\mu)\, d\varphi d\mu \equiv f_{nm}, \tag{5}$$

$$\Lambda_{nm}^{-1}[f_{nm}] = \frac{1}{\pi} \mathrm{Re} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \varepsilon_m \frac{f_{nm} e^{-im\varphi} P_n^m(\mu)}{\|P_n^m(\mu)\|^2} \equiv f(\varphi, \mu), \tag{6}$$

where $P_n^m(\mu)$ is the associated Legendre function of the first kind [2],

$$\varepsilon_m = \begin{cases} \frac{1}{2}, & m = 0 \\ 1, & m \geqslant 1 \end{cases}; \quad \|P_n^m(\mu)\|^2 = \frac{2(n+m)!}{(2n+1)(n-m)!} \text{ is the norm squared.}$$

Operator Λ_{nm} together with the Laplace integral operator L [3] facilitate reduction of the three-dimensional problem to a one-dimensional one.

Parameters $h_{ij}(i, j = \overline{1, 3})$ make it possible to extract from expression (4) the solutions to problem (1)-(3) for any combination of the first three kinds of boundary conditions stipulated at any of the surfaces $r = R_j$ ($j = 1, 2$), while the nonnegative arbitrary parameters b_0, b_1 make it possible to obtain purely undular ($b_1 \to 0$) as well as ordinary (parabola) ($b_1 = 1$, $b_0 \to 0$) temperature fields.

As an example the case where sphere D contains no heat sources ($f_1 = 0$) is considered, its initial temperature is zero ($f_2 = f_3 = 0$), its boundary $r = R_1$ is maintained at zero temperature, and its boundary $r = R_2$ is heated according to the law $\psi_2 = t_0 S_+(t)(1-\mu)^{-1/2}$, with $S_+(t)$ denoting the asymmetric unit function.

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TEMPERATURE FIELD OF A BURIED PIPELINE

B. A. Krasovitskii

UDC 536.242

The transport of many products over pipelines is associated with intensive heat exchange between the pipeline and the surrounding ground. The hydrodynamic characteristics of flow during pumping of heated crude oil or liquefied and cooled natural gas are most intimately related to the temperature of the stream. Pipeline transport of water, mixtures and suspensions on a water base, or other freezing fluids through cold grounds requires careful forecasting of the temperature field so that clogging of the pipeline can be prevented. The heat transfer processes are particularly intensive during the startup period, characterized by the largest temperature drops and appreciable nonsteadiness.

The magnitude of thermal flux passing to the ground determines the rate of temperature change in the product along the pipeline and depends on the temperature field of the ground around the pipeline. This temperature field is determined by two factors: perturbation-generating effect of the pipeline, whose temperature is generally different from the natural temperature of the ground, and periodic changes in the natural temperature of the ground due to seasonal fluctuations of the air temperature. Here the problem of thermal interaction between a pipeline and the surrounding ground, taking these factors into account, is formulated mathematically. The fundamental system of equations is simplified so that simple algorithms of its solution can be constructed. An approximate solution for the temperature field of the ground around a buried pipeline is found in the form combining the solutions to axisymmetric problems of thermoelasticity. The latter solutions are obtained by the integral method. In this way the expression for the thermal flux in the ground can be written in a closed form. For the purpose of estimating the accuracy of this solution, it is compared with the results of a numerical solution.

The numerical solution was obtained by conformal mapping of the given region into a unit square. The heat-conduction problem within the conformal region was solved by the locally uniform difference scheme. A comparison of the results indicates that the method proposed here gives an error not exceeding 6%.

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EFFECT OF THE STEFAN FLUX ON THE HYDRODYNAMICS AND THE HEAT TRANSFER INVOLVING AN EVAPORATING SPHERICAL PARTICLE IN MOTION

B. I. Abramzon, B. M. Abramzon, and G. A. Fishbein

UDC 532.516

A study is made of the role which the Stefan flux plays in evaporation of a single droplet in a stream blowing on its surface at a constant velocity or at a velocity not known beforehand but determined from the solution to the equation of convective diffusion.

The evaporation process is assumed to be quasisteady, the vapor near the surface to be saturated and its concentration to be a function of the droplet temperature only. The physical properties of the vapor–gas mixture near the droplet are assumed to be constant and to have been evaluated at some mean temperature and mean vapor concentration in the stream. Internal movement of the liquid within the droplet is assumed to have no effect on the external streamlining.

On the basis of these assumptions, the problem reduces to the following system of Navier–Stokes equations, equations of diffusion, and heat-transfer equations for the vapor–gas mixture

$$\frac{N_{Re}}{2}\left[\frac{\partial\psi}{\partial r}\frac{\partial}{\partial\theta}\left(\frac{E^2\psi}{r^2\sin^2\theta}\right)-\frac{\partial\psi}{\partial\theta}\frac{\partial}{\partial r}\left(\frac{E^2\psi}{r^2\sin^2\theta}\right)\right]\sin\theta=E^4\psi, \tag{1}$$

where

$$E^2=\frac{\partial^2}{\partial r^2}+\frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right),$$

$$\frac{N_{Re}N_{Sc}}{2}\left(v_r\frac{\partial Z}{\partial r}+\frac{v_\theta}{r}\frac{\partial Z}{\partial\theta}\right)=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial Z}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial Z}{\partial\theta}\right), \tag{2}$$

$$\frac{N_{Re}N_{Pr}}{2}\left(v_r\frac{\partial t}{\partial r}+\frac{v_\theta}{r}\frac{\partial t}{\partial\theta}\right)=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right). \tag{3}$$

Here ψ is the flow function; $Z = (C-C_0)/(C_1-C_0)$, relative concentration; C, relative mass concentration of vapor in the gaseous mixture; $t = (T-T_0)/(T_1-T_0)$, relative temperature; T, absolute temperature; N_{Re}, Reynolds number; N_{Pr}, Prandtl number; and N_{Sc}, Schmidt number. Subscript "0" refers to the droplet surface; subscript "1" refers to the oncoming stream.

The relation between the flow function and the blowing stream velocity at the surface of a sphere is

$$\psi=-\int_0^\theta V_R(\theta)\sin\theta\,d\theta. \tag{4}$$

The quantity V_R can be defined according to the expression

$$V_R=\frac{\sigma}{N_{Re}N_{Sc}}\,\mathrm{sh}_\theta,$$

where parameter $\sigma = (C_0-C_1)/(1-C_0)$ determines the intensity and the direction of the Stefan flux; sh_θ is the local mass-transfer coefficient.

The problem is solved by the finite-differences method for values of the Reynolds number $N_{Re} \leq 100$ with either uniform or nonuniform injection of mass at the droplet surface. The coefficients of friction and frontal drag are calculated as functions of N_{Re} and V_R. Determined are the characteristic features of the zone with reverse-vortical flow during injection and suction respectively. Obtained are also the values of local and mean heat and mass transfer coefficients, these values being compared with experimental data corresponding to a Stefan flux parameter within the $-0.5 \le \sigma \le 0.5$ range.

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EQUALIZATION OF A STREAM BY MEANS OF A HONEYCOMB

A. S. Mazo

UDC 532.555/56

Under consideration is the equalizing effect of a honeycomb with a uniform (over the cross section) drag on nonseparation flow of an incompressible fluid with an arbitrary initial velocity profile in a channel of uniform cross section. The problem is solved by the methods of hydraulics, on the basis of the following flow pattern. Along the inlet section 1 at some distance before the honeycomb and at section 2 immediately behind the honeycomb there is a constant static pressure and a zero transverse velocity. The flow rate and thus also the mean velocity along the honeycomb tubes remain constant so that the flow redistribution must occur before the honeycomb. The total pressure in the jet filaments is, meanwhile, assumed to remain constant. Considering the case of only a slight nonuniformity, the author solves the problem by the method of perturbations, accurately down to terms of second-order smallness. The stream is subdivided into n elementary jet filaments and for each of the latter is written the continuity equation as well as Bernoulli equation, with losses in each jet filament flowing through the honeycomb assumed to be proportional to the velocity in the honeycomb tubes squared (the velocity being equal to the velocity u_2 at the outlet). With a given velocity profile at the inlet and with the geometrical condition of constant channel area, this yields a closed system of algebraic equations.

An analysis of the equations in this approximation reveals that the deviation of velocity from the mean one $|u_i - u_0|$ decreases in the same proportion in each jet filament upon passage through the honeycomb. The magnitude of the nonuniformity factor ψ_2 behind the honeycomb does not depend on the shape of the initial velocity profile, but is related to the initial nonuniformity factor ψ_1 and the drag coefficient ζ in the honeycomb according to the expression

$$\psi_2 = \frac{\psi_1}{1+\zeta}$$

for the case of square-law drag (ζ = const). Complete equalization of a stream is thus possible only when $\zeta \to \infty$. This conclusion disagrees fundamentally with the well-known concept about meshes, where complete equalization occurs when $\zeta = 2$ and the velocity profile becomes inverted when $\zeta > 2$. Physically this is attributable to the fact that during passage through a mesh (unlike through a honeycomb) the incidence angle does not change and, therefore, the drag here remains proportional to the square of the arithmetic mean of the velocities before and after equalization.

In the case of a linear drag law, corresponding to laminar flow, equalization of a stream proceeds according to the relation

$$\psi_2 = \frac{\psi_1}{1+0.5\zeta_0}$$

(ζ_0 denoting the drag coefficient in a uniform stream with the same flow rate) so that producing the same equalization effects requires a "laminar" honeycomb with double the drag of a "turbulent" one.

An array of honeycombs one behind another makes equalization feasible with lower drag losses.

These theoretical results have been compared with experimental data for the case of square-law drag. A close agreement was found between the calculated and the measured dependence of the nonuniformity factor on the honeycomb drag.

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CALCULATION OF SURFACE FRICTION ALONG INITIAL PIPE SEGMENTS IN A TURBULENT BOUNDARY LAYER WITH WHIRLING OF THE STREAM AT THE INLET

V. M. Sobin

UDC 532.517.4

An approximate method of calculating the surface friction in a turbulent boundary layer of an incompressible fluid with whirling of the stream at the inlet is shown here.

The problem is solved with the aid of the integral method [1], which happens to be conservative with respect to the exact velocity profile when the boundary region is correctly simulated. The equations of motion within the boundary layer, written in a cylindrical system of coordinates with appropriate boundary conditions, are reduced to a system of two nonlinear ordinary differential equations of the first order with respect to parameters λ and s of surface friction. For the axial component of velocity in the boundary layer is used the universal logarithmic profile $u^+ = 2.5 \log y^+ + 5.5$ and for the tangential component of velocity is used the profile $-w^+ = u^+ s$. Here s is regarded as a function of x only. After estimating the magnitudes of the terms in the resulting equations, the latter can be simplified to

$$\delta^+ (\lambda^2 - 5\lambda + 12{,}5) \frac{d\lambda}{d\bar{x}} + \frac{\bar{u}_0'}{\bar{u}_0} 5\delta^+ \lambda (\lambda - 2.5) = N_{Re} \bar{u}_0. \tag{1}$$

With the approximations

$$\begin{aligned} \delta^+ (\lambda^2 - 5\lambda + 12.5) &= 1.9 \exp(0.545\lambda), \\ 5\delta^+ \lambda (\lambda - 2.5) &= 11.1 \exp(0.545\lambda) \end{aligned} \tag{2}$$

Eq. (1) admits an analytical solution.

It is demonstrated that in many practical cases the velocity $\bar{u}_0$ = const. Taking this into account, the equation for s becomes

$$\frac{ds}{d\bar{x}} + \left[\frac{N_{Re} \bar{u}_0}{\delta^+ \lambda (\lambda^2 - 5\lambda + 12.5)} - \frac{1}{\lambda} \frac{d\lambda}{d\bar{x}} \right] s = 0 \tag{3}$$

and can, with the use of the first approximation (2) be easily integrated.

Finally, for c_f and s are obtained the explicit expressions

$$c_f = \frac{0.594}{\ln^2 \left(\frac{z_0}{\bar{x}_0} \bar{x} \right)}, \tag{4}$$

$$s = s_0 \frac{c_f}{c_{f_0}}, \tag{5}$$

with the constants corrected on the basis of experimental data.

It is noteworthy that profile (4) describes closely also the distribution of the total coefficient of surface friction and profile (5) very accurately predicts self-adjointness of whirling angles in the stream with respect to the Reynolds number.

NOTATION

$\bar{x} = x/d$	is the dimensionless longitudinal coordinate;
u_0	is the value of velocity u at the edge of the boundary layer;
s	is the tangent of the whirling angle relative to the pipe axis within the immediate vicinity of the wall;

τ_{10} is the axial component of shearing stress at the wall;
x_0, s_0, and λ_0 are the values of x, s, and λ at the inlet section;
$u^+ = u/u_*$; $y^+ = yu_*/\nu$; $u_* = \sqrt{\tau_{10}/\rho}$; $c_f = 2\tau_{10}/\rho u_0^2$; $\lambda = u_0/u_* = (2/c_f)^{1/2}$; $\bar{u}_0 = u_0/u_{av}$;
$\delta^+ = \exp[(\lambda - 5.5)/2.5]$;
$z_0 = \exp(0.545\lambda_0)$;
$N_{Re} = u_{av}d/\nu$.

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VELOCITY FIELD INSIDE A CYLINDRICAL VESSEL WITH A ROTARY STIRRER AT THE BOTTOM

Yu. V. Martynov

The flow of a liquid inside a cylindrical vessel with a rotary stirrer near the bottom is analyzed in the case of a small ratio of cylinder radius to liquid level in the vessel and a stirrer with many blades, assuming that the funnel forming as a result of intensive stirring reaches the bottom. The stirrer blades are extended toward the bottom so that, owing to the small clearance between stirrer and bottom, there will occur only insignificant changes in the stream. The funnel is approximated with a cone. According to data in [1], the stream of liquid flows from underneath the stirrer radially outward and at the wall turns vertically upward, whereupon it is sucked in by the stirrer. Considering that the concave surface of the vessel has a stabilizing effect on the stream [2] and that no turbulence of the stream occurs at the free surface, one can assume the flow in the mixer vessel to be nonturbulent (except within a small region around the stirrer). First the flow in the entire region is calculated, whereupon the region with high turbulence is removed. The equation for the flow function describing an axisymmetric stream of a nonviscous incompressible fluid is given elsewhere [3], in a system of spherical coordinates with the origin at the vertex of the funnel cone, and zero boundary values are stipulated at the cone axis as well as at the cylinder surface. Furthermore, following the procedure in [3], it is assumed that $\Phi(\psi) = (k\psi^2 + 2c\psi + b)^{1/2}$, $F(\psi) = a_0 + a_1\psi + a_2\psi^2/2$, and k, c, a_0, a_1, a_2, b = const. After a change of variables $z = \xi^2\sin^2\vartheta$, $x = \cos^2\vartheta - \cos\vartheta_1\cos\vartheta$, expansion of the coefficients of the derivatives into Taylor series, with small terms disregarded inasmuch as $\cos\vartheta_1 \approx 1$, reduces the problem to

$$4z\frac{\partial^2\psi}{\partial z^2} + [5x^2 - 4(5\varepsilon - 2)x]\frac{\partial^2\psi}{\partial z\partial x} + x^2(16\varepsilon + 4)\frac{1}{z}\frac{\partial^2\psi}{\partial x^2} + \frac{4x^2}{z}\frac{\partial\psi}{\partial x} + k\psi + C + za_2\psi + za_1 = 0,$$

$$\psi(z, 0) = \psi(1, x) = \psi(0, x) = 0,$$

$$v_\varphi = \Phi(\psi)/(\xi\sin\vartheta), \quad v_\xi = (\xi^2\sin^2\vartheta)^{-1}\partial\psi/\partial\vartheta,$$
$$v_\vartheta = -(\xi\sin v)^{-1}\partial\psi/\partial\xi.$$

Its solution is sought in the form of a Taylor series $\psi = \sum_{n=1}^{\infty} x^n f_n(z)/n!$, with $f_0(z) = 0$, since the boundary conditions make $\psi(0, x) = 0$. After the real part has been extracted, the solution for the first term of that series (all other terms being relatively small) is written as $\psi(\vartheta, \xi) = \cos\vartheta(\cos\vartheta - \cos\vartheta_1)C_1N_{Re}\{\exp(i\sqrt{a_2}\xi^2\sin^2\vartheta/2)\Phi(1 - (5/3)\varepsilon + k/(4i\sqrt{a_2}),\ 2-5\varepsilon;\ -i\sqrt{a_2}\xi^2\sin^2\vartheta)\}$. Here $\Phi(a, b, z)$ is the Kummer function. The constants k, a_2, c_1, b are determined from the system of transcendental equations $\Phi(1-(5/3)\varepsilon + k/(4i\sqrt{a_2}),\ 2-5\varepsilon;\ -i\sqrt{a_2}) = 0$; $\partial\psi(\vartheta_3, \xi_2)/\partial\vartheta = 0$ (the first one being introduced so as to satisfy the second boundary condition, the second one valid

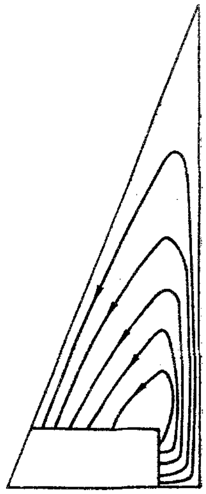

Fig. 1. Equidistant lines of secondary flow, plotted in 0.01 steps beginning from 0.01 (flow line nearest to the free surface).

inasmuch as the jet stream flowing from underneath the stirrer is axisymmetric) and the system of algebraic equations $(k\psi^2(\vartheta_4, \xi_6) + b)^{1/2} = 1$; $v_\xi(\vartheta_5, \xi_7)/v_\varphi(\vartheta_5, \xi_7) = \alpha$ (the first one equating the azimuthal velocities of blades and liquid near the stirrer, the second one quantitatively relating the azimuthal motion and the axial motion). The value of α is either determined experimentally or taken from published sources; it depends on stirrer and vessel design parameters. For choosing the necessary root of the system of transcendental equations there is derived an estimating equation. It is obtained by equating the arithmetic mean velocity, calculated on the basis of four velocities (at the tip of a stirrer blade, at the center of a lateral surface, at the free surface near the stirrer, and at the upper level of the liquid) to $v_+ = \int_S (k\psi^2 + b)^{1/2}/(\xi \sin\vartheta)ds/S = 1.69$

$$(kQ^2(8\pi^2)^{-1} + b)^{1/2} = v_+ = \xi_5^2(1/\xi_5 + 2 + 1/\xi_9)/4.$$

On the basis of the expressions derived here, a calculation is made of the velocity field in a mixer vessel with the dimensions $\vartheta = 21^0$; $2\xi_8 = 2.4$; $\xi_3 = 0.3$; $\xi_5 = 0.8$, as shown in Fig. 1. A numerical solution of the system of transcendental and algebraic equations yielded the values $a_2 = -4.365$, $k = 0.609$, $c_1 = 0.302$, $b = 0.998$, with 1.06 having been the estimated value of b.

NOTATION

ψ	is the flow function;
C_1	is the constant;
ϑ	is the axial angle;
ξ	is the radius;
$\Phi(\psi)$	is the circulation;
$F(\psi)$	is the Bernoulli constant;
v_ξ, v_φ, v_ϑ	are, respectively, the radial, azimuthal, and axial components of velocity;
ξ_1	is the blade width;
ξ_5	is the blade length;
ξ_8	is the half-height of the liquid level;
v_+	is the arithmetic mean azimuthal velocity;
ξ_9	is the projection of the point where funnel surface and stirrer surface intersect on the vessel axis;
Q	is the rate of secondary flow of the liquid;
a_0, a_1, a_2, k, c, b, α	are the constants;
$\xi_1 = 1$	is the vessel radius;
$v^*\varphi = 1$	is the azimuthal angular velocity of the tip of a stirrer blade;
$\xi_4 = \xi_5 + (1-\xi_5)/2$;	
$\varepsilon = 1-\cos^2\vartheta_1$; $\xi_2 = (\xi_3^2 + \xi_4^2)^{1/2}$;	

$\xi_6 = (\xi_5 + \xi_3^2/4)^{1/2}$; $\vartheta_2 = \pi/2$; $\vartheta_4 = \arctan\,[\xi_3/(2\xi_5)]$;
$\vartheta_5 = \arctan \xi_8$;
S is the cross-sectional area of the liquid in the mixer vessel in the plane of a radius and an axial angle.

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HEATING OF A MATERIAL BY A SURFACE SOURCE AND AN INTERNAL SOURCE OF HEAT

V. M. Kulyapin and A. I. Pechenkin

UDC 536.248.2

Under consideration is the one-dimensional nonlinear problem of fusion and evaporation of a material by a surface source and an internal source of high-density thermal flux. The temperature profile in the liquid phase satisfies the equation of heat balance and, according to the analysis in [1], is

$$\vartheta_1(x,\ t) = T_0 + \frac{T_m - T_0}{X - X_0} + \left(\frac{qv}{2\lambda_1} - \frac{1}{2a_1}\frac{dT_0}{dt}\right)[(X - X_0)(x - X_0) - (x - X_0)^2]. \tag{1}$$

The trend of processes occurring here will vary depending on the relation between surface source and internal source. The width $X - X_0$ of the molten zone is

$$X - X_0 = -\frac{\varepsilon}{q} U(t^*). \tag{2}$$

From the subsequently given equations will follow that $U(t^*) < 0$.

Evolution of heat within the molten volume,

with $|\alpha| < 0.25$

$$t^* = \frac{1}{2|\alpha|}\ln(|\alpha| U^2 + U + 1) - \frac{1}{2|\alpha|\sqrt{1-4|\alpha|}}\left(\ln\frac{2|\alpha| U + 1 - \sqrt{1-4|\alpha|}}{2|\alpha| U + 1 + \sqrt{1-4|\alpha|}} - \ln\frac{1-\sqrt{1-4|\alpha|}}{1+\sqrt{1-4|\alpha|}}\right); \tag{3}$$

with $|\alpha| > 0.25$

$$t^* = \frac{1}{2|\alpha|}\ln(|\alpha| U^2 + U + 1) - \frac{1}{|\alpha|\sqrt{4|\alpha|-1}}\left(\operatorname{arctg}\frac{2|\alpha| U + 1}{\sqrt{4|\alpha|-1}} - \operatorname{arctg}\frac{1}{\sqrt{4|\alpha|-1}}\right). \tag{4}$$

Absorption of heat within the molten volume, with $\alpha > 0$

$$t^* = \frac{1}{2\alpha\sqrt{1+4\alpha}}\left(\ln\frac{\sqrt{1+4\alpha} + 2\alpha U - 1}{\sqrt{1+4\alpha} - 2\alpha U + 1} - \ln\frac{\sqrt{1+4\alpha} - 1}{\sqrt{1+4\alpha} + 1}\right) - \frac{1}{2\alpha}\ln(1 + U - \alpha U^2), \tag{5}$$

where

$$\alpha = \pm\frac{\varepsilon q v}{2q^2}\left[\frac{L_0}{L + c(T_0 - T)} - 1\right];$$

$$\varepsilon = \lambda_1(T_0 - T_m)\left[1 + \frac{L_0}{L + c(T_0 - T)}\right];\ t^* = \frac{q^2 t}{L_0 \rho \varepsilon}.$$

It is evident, according to expressions (3)-(5), that the molten zone stabilizes during absorption of heat within its volume and during evolution of heat with $|\alpha| < 0.25$. As the internal source increases, the fusion process becomes nonsteady.

NOTATION

q	is the surface density of thermal flux, W/cm^2;
q_V	is the volume density of thermal flux, W/cm^3;
$X_0(t)$	is the breakdown boundary, cm;
$X(t)$	is the fusion boundary, cm;
$X - X_0$	is the molten zone, cm;
x	is the running space coordinate, cm;
T_0, T_m, $\overline{T}$	are, respectively, the temperature of the breakdown surface, melting point, and initial temperature, °K;
t	is the time, sec;
λ_1	is the thermal conductivity of the liquid material, W/cm · deg C;
a_1	is the thermal diffusivity of the liquid material, cm^2/sec;
c	is the mean specific heat, J/g · deg C;
L_0	is the latent heat of evaporation;
L	is the latent heat of fusion;
ρ	is the density, g/cm^3;
ϑ	is the temperature profile in the liquid phase.

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SOLUTION OF THE THREE-DIMENSIONAL TRANSIENT PROBLEM OF HEAT CONDUCTION IN BODIES OF INTRICATE SHAPES

V. M. Kapinos and Yu. L. Khrestovoi

UDC 621.165

Determination of the temperature fields in bodies of intricate shapes reduces to a solution of the three-dimensional transient problem of heat conduction for irregular regions. This is often done by grid methods. When problems for irregular regions are solved with the aid of equations which have been derived for regular (rectangular, cylindrical, spherical) regions, then difficulties arise in referring the boundary conditions to grid points on the boundary.

It is, therefore, proposed to transform an irregular region to a regular one by a linear change of coordinates which, on the one hand, will eliminate attrition of the boundary conditions and on the other hand will establish conditions making it easy to construct algorithms of temperature field calculations for bodies of intricate shapes.

The heat-conduction equations and the appropriate boundary conditions, in a system of cylindrical coordinates, for a region bounded by the intersection of surfaces $r = R(z, \varphi)$ and $r = r_0(z, \varphi)$ which have continuous first derivatives with planes $z = 0$, $z = l$, $\varphi = 0$, $\varphi = \Pi$ at $R > r_0$ have, accordingly, been transformed by the change

$$x_1 = a + b\frac{r - r_0}{R - r_0}, \; x_2 = z, \; x_3 = \varphi,$$

where a and b are determined from the conditions $r = r_0$, $x_1 = a$; $r = R$, $x_1 = a_1$, $b = a - a_1$.

Therefore, a and a_1 are, respectively, the inside radius and the outside radius of the reference region constituting a part of a straight hollow cylinder of length l contained between two radial planes at angle Π to each other.

The new system of coordinates is not orthogonal with a nonzero Jacobian.

The heat-conduction equation in these coordinates is

$$\frac{1}{a}\frac{\partial t}{\partial \tau} = B_1\frac{\partial^2 t}{\partial x_1^2} + B_2\frac{\partial t}{\partial x_1} + B_3\frac{\partial^2 t}{\partial x_2^2} + B_4\frac{\partial^2 t}{\partial x_1 \partial x_2} + B_5\frac{\partial^2 t}{\partial x_3^2} + B_6\frac{\partial^2 t}{\partial x_1 \partial x_3},$$

where

$$B_1 = A^2 + A_1^2 + A_2^2/r^2;\ B_2 = \frac{A}{r} + \frac{\partial A_1}{\partial Z} + \frac{1}{r^2}\frac{\partial A_2}{\partial \varphi};\ B_3 = 1,\ B_4 = 2A_1;$$

$$B_5 = 1/r^2,\ B_6 = 2A_2/r^2;\ A = \frac{\partial x_1}{\partial r},\ A_1 = \frac{\partial x_1}{\partial z},\ A_2 = \frac{\partial x_1}{\partial \varphi}.$$

The boundary conditions, after transformations, become

$$-\frac{\alpha_b}{\lambda}(t - t_f) = \left[A\cos(n, r) + A_1\cos(n, z) + \frac{A_2}{r}\cos(n, \varphi)\right]\frac{\partial t}{\partial x_1} + \cos(n, z)\frac{\partial t}{\partial x_2} + \cos(n, \varphi)\frac{1}{r}\frac{\partial t}{\partial x_3},$$

with the heat-transfer coefficient α_b and the temperature of the medium t_f both being functions of time as well as of the coordinates, with λ denoting the thermal conductivity, a denoting the thermal diffusivity, and n denoting the outward normal.

The resulting system is solved numerically according to the efficient additive scheme with a floating weight.

The region of the temperature field determination is described as follows. The body is subdivided into segments by radial planes and planes perpendicular to the z axis. The outside surface and the inside surface are described by a conical surface or a plane surface. The grid is constructed so that the calculation points lie only on the region boundaries and never on the segment boundaries.

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ROLE OF CAVITATION IN THE ULTRASONIC CAPILLARY EFFECT

V. G. Barantsev and V. N. Motorin

UDC 534.14

A series of experiments was performed for the purpose of studying the effect of ultrasound on the capillary rise of liquids. Particular attention was paid to physical factors causing the pressure head to increase under the influence of ultrasound.

An experimental apparatus was set up for this which included a system of measurements with recording of the instantaneous velocity of the liquid along the capillary on a motion picture film. The experiments were performed under the following conditions: radiator power N = 25 W, vibration frequency f = 41 kHz, diameters of the capillaries d = 0.21, 0.61, 1.16, 2.0, 3.0 mm. As the active medium were used water, ethyl alcohol, and transformer oil. The amplitude of sound pressure in the liquid was measured with a hydrophone. Test results are shown for $P_0 = 1.58$ atm.

The results of these experiments revealed the trend of changes in the velocity of capillary rise with time, depending on the tube diameter and on the kind of liquid. The rise velocity is dependent on the distance from the radiator to the base of the capillary. The experimental data are presented in the form of graphs $h = f(\tau)$, τ denoting the interval of time within which the liquid level had risen by the height h. At the same time was also measured the maximum rise of the liquid level depending on the tube diameter. In the capillary with a diameter of 3.0 mm it was found to be 105.4 times higher than the equilibrium rise of liquid without ultrasound.

Cavitation in the liquid was observed visually and recorded on photographic film through a microscope with a camera attachment. In all the experiments the velocity of capillary rise was found to become highest with the base of the capillary placed in a cavitation cloud or directly above it. The readings also became stable under this condition.

The phenomenon of cavitation was observed only at ultrasound intensities above the threshold level. As the amplitude of acoustic waves dropped below that threshold, cavitation in the liquid ceased and the liquid column dropped to the equilibrium height of capillary rise without ultrasound.

The results of these experiments confirm the conclusion that acoustic cavitation in the liquid is the cause of the ultrasonic capillary effect.

The authors propose a physical model which, in their view, explains the process of capillary rise of a liquid under the influence of ultrasound, namely some truncation of the normal amplitude of ultrasonic waves during the rarefaction phase as a result of ultrasonic cavitation in the liquid. It is suggested that acoustic cavitation in a liquid results in some truncation of the amplitude of ultrasonic waves during the rarefaction phase. The truncation level is determined by an empirical coefficient ($\varkappa$) which depends on the conditions of the experiment: diameter of the capillary tube, kind of liquid, etc.

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GENERAL SOLUTION TO THE THERMOELASTICITY PROBLEM FOR AN ASYMMETRICALLY HEATED SOLID CYLINDER

A. G. Sabel'nikov, K. I. Skorikov, and A. N. Strigunov

UDC 539.377

Under consideration is an infinitely long solid cylinder with the radius R asymmetrically heated in the circumferential direction.

Its state of thermal stress is defined by the stress function $\Phi = \Phi(r, \varphi, \tau)$, which is the general solution to the nonhomogeneous biharmonic equation

$$\Delta(\Delta\Phi) = \frac{1+\mu}{1+\mu}\Delta\beta t. \tag{1}$$

For determining the components of the stress tensor $\sigma_{ij}(i, j = r, \varphi, z)$, the temperature field $t = t(r, \varphi, \tau)$ is represented as the sum of two components:

$$t(r, \varphi, \tau) = t_1(r, \tau) + t_2(r, \varphi, \tau). \tag{2}$$

Here $t_1(r, \tau)$ is the symmetric component of the temperature field and $t_2(r, \varphi, \tau)$ is the asymmetric component.

The stresses calculated according to Eq. (1) will then appear as the sum of two solutions to the thermoelasticity problem, one of them corresponding to $t_1(r, \tau)$ and found by conventional methods. The other solution, corresponding to $t_2(r, \varphi, \tau)$, is determined from the thermoelastic displacement potential $\Phi = \Phi(r, \varphi, \tau)$, which satisfies the equation

$$\Delta\bar{\Phi} = \frac{1+\mu}{1-\mu}\beta t_2(r, \varphi, t), \tag{3}$$

and from the general solution to Eq. (1) without the right-hand side. The components of the stress tensor are expressed in dimensionless form. From the general expressions given here can be obtained those for special cases which correspond to symmetric heating of an infinitely long solid cylinder and which are identical to the well-known expressions for these cases.

An example is considered to illustrate the practical application of these solutions. It is based on experimental data on heating of solid cylindrical billets by impinging jets in a high-speed convection furnace.

NOTATION

r	is the radial coordinate, m;
φ	is the angular coordinate;
τ	is the time, sec;
μ	is the Poisson's ratio;
β	is the coefficient of thermal expansion, 1/deg C;
$\Delta = (\partial^2/\partial r^2) + (1/r) \cdot (\partial/\partial r) + (1/r^2) \cdot (\partial^2/\partial\varphi^2)$	is the Laplace operator.

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